IMPROVED CROSSOVER FILTERS AND METHOD

BACKGROUND OF THE INVENTION

The present invention relates to crossover filters suitable for dividing wave propagated phenomena or signals into at least two frequency bands.

The phenomena/signals are to be divided with the intention that recombination of the phenomena/signals can be performed without corrupting amplitude integrity of the original phenomena/signals.

The present invention will hereinafter be described with particular reference to filters in the electrical domain. However, it is to be appreciated that it is not thereby limited to that domain. The principles of the present invention have universal applicability and in other domains, including the electromagnetic, optical, mechanical and acoustical domains. Examples of the invention in other domains are given in the specification to illustrate the universal applicability of the present invention.

Crossover filters are commonly used in loudspeakers which incorporate multiple electroacoustic transducers. Because the electroacoustic transducers are designed or dedicated for optimum performance over a limited range of frequencies, the crossover filters act as a splitter that divides the driving signal into at least two frequency bands.

The frequency bands may correspond to the dedicated frequencies of the transducers. What is desired of the crossover filters is that the divided frequency bands may be recombined through the transducers to provide a substantially accurate representation (ie. amplitude and phase) of the original driving signal before it was divided into two (or more) frequency bands.

Common shortcomings of prior art crossover filters include an inability to achieve a recombined amplitude response which is flat or constant across the one or more crossover frequencies and/or an inability to roll off the response to

each electroacoustic transducer quickly enough, particularly at the low frequency side of the crossover frequency. Rapid roll off is desirable to avoid out of band signals introducing distortion or causing damage to electroacoustic transducers. Prior art designs achieve rapid roll off by utilizing more poles in the filter design since each pole contributes 6dB per octave additional roll off. However a disadvantage of this approach is that it increases group delay. An object of the present invention is to alleviate the disadvantages of the prior art.

SUMMARY OF THE INVENTION

The present invention proposes a new class of crossover filters suitable for, inter alia, crossing over between pairs of loudspeaker transducers. The crossover filters of the present invention may include a pair of filters such as a high pass and a low pass filter. Each filter may have an amplitude response that may include a notch or null response at a frequency close to or in the region of the crossover frequency. A notch or null response above the crossover frequency in the low pass filter and below the crossover frequency in the high pass filter may provide a greatly increased or steeper roll off for each filter of the crossover for any order of filter. Notwithstanding the notch or null response the amplitude responses of the pair of filters may be arranged to add together to produce a combined output that is substantially flat or constant in amplitude at least across the region of the crossover frequency. Benefits of such an arrangement include improved amplitude response and improved out of band signal attenuation close to the crossover frequency for each band.

It may be shown that the transfer function of the summed output of nth order crossover filters wherein each filter incorporates a second order notch is

$$F(sT_X)_{\Sigma n} = \frac{\overbrace{(1 + k^2s^2T_X^2)}^{\text{LOW-PASS}} \pm \overbrace{s^{n-2}T_X^{n-2}(k^2 + s^2T_X^2)}^{\text{HIGH-PASS}}}{F_{DENn}(sT_X)} \qquad - (1)$$

where k is the ratio of lower notch frequency f_{NL} in the high-pass response to the crossover or transition frequency f_X

$$k = f_{NL}/f_X = f_X/f_{NH} \qquad - (2)$$

and where f_{NH} is the higher notch frequency in the low-pass response, and T_X is the associated time constant of the crossover frequency ($T_X = 1/2\pi f_X$). The present invention is applicable to notches of higher order but second order notches are sufficient to illustrate the principle.

The common denominator $F_{DENn}(sT_X)$ is derived from the numerator of the summed response by factorising it into first and second order factors, changing the signs of any negative first order terms in those factors to positive and then re-multiplying all the factors together. The summed response thus becomes an all-pass function whose numerator is the product of all the factors of the original numerator with negative first order terms.

According to one aspect of the present invention there is provided an improved filter system including a low pass filter having a response which rolls off towards a crossover frequency and a high pass filter having a complementary response which rolls off towards said crossover frequency such that the combined response of said filters is substantially constant in amplitude at least in the region of said crossover frequency, wherein said response of said low pass filter is defined by a low pass complex transfer function having a first numerator and a first denominator and said response of said high pass filter is defined by a high pass complex transfer function having a second numerator and a second denominator and wherein said second denominator is substantially the same as said first denominator and the sum of said first and second numerators has substantially the same squared modulus as said first or second denominator.

The low pass filter may include a first null response at a frequency in the region of and above the crossover frequency. The first null response may be provided by at least one complex conjugate pair of transmission zeros such that their imaginary parts lie in the stop band of the low pass transfer function within the crossover region. The high pass filter may include a second null response at a frequency in the region of and below the crossover frequency. The second null response may be provided by at least one complex conjugate pair of

transmission zeros such that their imaginary parts lie in the stop band of the high pass transfer function within the crossover region.

According to a further aspect of the present invention there is provided a method of tuning a filter system including a low pass filter having a response which rolls off towards a crossover frequency and a high pass filter having a complementary response which rolls off towards said crossover frequency such that the combined response of said filters is substantially constant in amplitude at least in the region of said crossover frequency, said method including the steps of: selecting a filter topology capable of realizing a low pass complex transfer function defined by a first numerator and a first denominator; selecting a filter topology capable of realizing a high pass complex transfer function defined by a second numerator and a second denominator; setting the second denominator so that it is substantially the same as the first denominator; and setting the squared modulus of the sum of the first and second numerators so that it is substantially the same as the squared modulus of the first or second denominator.

The method may include the step of determining coefficients for the transfer functions and the step of converting the coefficients to values of components in the filter topologies.

The invention may be realised via networks of any desired order depending upon the desired rate of rolloff for the resultant crossover. The invention may be realised using passive, active or digital circuitry or combinations thereof as is known in the art. Combinations may include but are not limited to an active low pass and passive high pass filter pair of any desired order, digital low pass and active high pass filter of any desired order, passive low pass and passive high pass filter of any desired order, digital low pass and digital high pass filter of any desired order, and active low pass and digital high pass filter realisations.

The invention may be further realised wherein the filter response is produced with a combination of electrical and mechano-acoustic filtering as may be the case where the electroacoustic transducer and/or the associated acoustic enclosure realise part of the filter response.

DESCRIPTION OF THE DRAWINGS

Preferred embodiments of the present invention will now be described with reference to the accompanying drawings wherein:

Fig. 1 shows generalised responses of even order notched high-pass and low-pass filters;

Fig. 2 shows a schematic circuit diagram for sixth order active high pass and low pass filters;

Fig. 3a shows the amplitude response for the low pass filter in Fig. 2;

Fig. 3b shows the phase response for the low pass filter in Fig. 2;

Fig. 4a shows the amplitude response for the high pass filter in Fig. 2;

Fig. 4b shows the phase response for the high pass filter in Fig. 2;

Fig. 5a shows the summed amplitude response for the low and high filters in Fig. 2;

Fig. 5b shows the summed phase response for the low and high pass filter in Fig. 2;

Fig. 6 shows responses of fourth order notched high-pass and low-pass filters;

Fig. 7 shows group delay responses for filters crossing over at 1 kHz;

Fig. 8 shows phase responses of fourth order ($k = 0.5774$) low-pass (upper) and high-pass (lower) filters;

Fig. 9. shows a Sallen & Key active filter incorporating a bridged-T network;

Fig. 10 shows a Sallen & Key active low-pass filter;

Fig. 11 shows a Sallen & Key active high-pass filter;

Fig. 12(a) shows a passive fourth-order low-pass filter (first kind);

Fig. 12(b) shows a passive fourth-order high-pass filter (first kind)

with components transformed $CnH = T_X^2 / LnL$ & $LnH = T_X^2 / CnL$ from Fig 12(a);

Fig. 12(c) shows a passive fourth-order high-pass filter (first kind) with inductances the result of Δ-Y transformation from Fig 12(b);

Fig. 12(d) shows a passive fourth-order high-pass filter (first kind) with inductances of Fig 12(c) realised as a coupled pair (series opposing);

Fig. 13(a) shows a passive fourth-order low-pass filter (second kind);

Fig. 13(b) shows a passive fourth-order low-pass filter (second kind) with inductances of Fig 13(a) realised as a coupled pair (series opposing);

Fig. 13(c) shows a passive fourth-order high-pass filter (second kind);

Fig. 13(d) shows a passive fourth-order high-pass filter (second kind);

Fig. 14 shows normalised input resistances and reactances of passive fourth-order filters with $k = 0.5774$ ($k^2 = {}^1/_3$): typical of all fourth-order notched crossovers;

Fig. 15 shows normalised input resistances and reactances of third-order passive filters for Butterworth crossovers;

Fig. 16 shows normalised input resistances and reactances of fourth-order passive filters for Linkwitz-Riley crossovers (equivalent to notched crossovers with $k = 0$); and

Fig. 17 shows an analog in the acoustical domain of the low-pass and high-pass filters shown in Figs. 13(a) and 13(b).

DESCRIPTION OF PREFERRED EMBODIMENTS

The generalised responses of even-order notched crossovers are shown in Fig 1. F_{NL} is the lower null centre frequency for the high pass filter, F_{NH} is the upper null centre frequency for the low pass filter, F_{PEAKH} is the upper peak frequency for the low pass filter, F_{INNERL} is the highest frequency at which the output of the high pass filter equals the peak value below the null for the high pass filter, F_{INNERH} is the lowest frequency at which the output of the low pass filter equals the peak value above the null for the low pass filter and F_X is the crossover or transition frequency. The in-band response of each filter rises at first to a small peak at the frequency of the out-of-band peak of the other filter. It then falls

back to reference 0dB level at the other filter's notch frequency, and onwards to -6.0dB at the transition frequency f_X.

The response falls to a null at its f_N, then rises to dB_{PEAK} at f_{PEAK} before falling away again at extreme frequencies at a rate, for an nth order filter, of 6(n-2)dB per octave. The effective limit of its response is at f_{INNER} where it has first passed through dB_{PEAK}.

Figure 2 shows the schematic circuit diagram for a sixth order active circuit embodiment of the invention. In this figure the low pass filter includes IC2, IC3 and IC4 and the high pass filter includes IC5, IC6 and IC7. An inverter, ICI is provided between the low and high pass filters to correct phase for the signals. IC3 and associated network generate the required second order filter transfer function for the low pass filter and IC2 and associated network generate two single order cascaded section responses as required. IC4 realises the notch in the low pass filter utilising Sallen & Key topology as known in the art. IC7 realises the notch in the high pass filter also utilising Sallen & Key topology as known in the art. IC6 and associated network generate the required second order filter transfer function for the high pass filter and IC5 and associated network generate two single order cascaded section responses as required. The filter sections use Sallen & Key topology as known in the art. The outputs of IC4 and IC7 provide signals to the low and high frequency electroacoustic transducers respectively. Inspection of signals in this network will reveal the response curves shown in figures 3, 4 and 5.

The solid curves of Fig 6 are for notched responses with k^2 figures of $^1/_3$, $^1/_4$ and $^1/_5$. The dashed curves, for comparison, are for Linkwitz-Riley responses of second order (upper) and fourth order (lower), with the same crossover frequency. In all cases, the notched response first reaches the level of dB_{PEAK} at f_{INNER}, while the Linkwitz-Riley response reaches it near f_{PEAK}, which is more than 1.5 times (0.6 octave) further away.

Beyond the notches, the fourth order responses eventually run parallel to the second order Linkwitz-Riley response, but k^2 times lower, i.e. by 9.5dB, 12.0dB or 14.0dB.

In Fig 7, the solid curves of group delay for the same notched responses are compared with the dashed curves for Linkwitz-Riley responses of fourth order (upper) and second order (lower). The curves are for a crossover frequency of 1kHz. For other crossover frequencies, the frequencies can be scaled in proportion, while the group delays are scaled in inverse proportion to the crossover frequency. *The curves apply equally to low-pass, high-pass and summed outputs.*

The transfer functions of the low-pass, high-pass and summed outputs of these even-order crossovers have numerators whose terms are all of even order. Thus they make no contribution to the group delay, and since all have the same denominator, the one curve of group delay applies to all.

In Fig 8, the curves of phase difference between input and output for the low-pass and high-pass filters are parallel at all frequencies. They are a constant 360° apart at all frequencies between the notches and 180° apart at all frequencies beyond.

The results presented in Figs 6, 7 & 8 for fourth order notched responses with $k^2 = {}^1/_3$ may be taken as generally typical of other even order notched responses with different values of k^2.

The responses of the odd-order functions are similar to those of even order, except that, because the individual high- and low-pass outputs combine in quadrature, each is now down to -3.0dB, instead of -6.0dB, at the crossover frequency f_X. The individual outputs now have a constant phase difference of 90° at frequencies between the two notches. At frequencies beyond, the inversion of polarity leaves the two outputs to still add in quadrature. Thus the in-band responses now *fall* initially, by less than 0.01dB, before rising to

reference level and then falling again to the stop band, in the manner of odd order elliptic function filters.

It turns out, not surprisingly, that when k is zero, so that the notch frequencies move outwards to zero and infinite frequencies, the transfer functions degenerate into Butterworths for odd order functions and double Butterworths [*A.N. Thiele – Optimum passive loudspeaker dividing networks – Proc. IREE Aust, Vol 36, No 7, July 1975, pp. 220-224*] (i.e. Linkwitz-Rileys [*S.H. Linkwitz – Active crossover networks for non-coincident drivers – JAES. Vo. 24. No.1, January/February 1976, pp.2-8 and in Audio Engineering Society, Inc, New York, October 1978, pp. 367-373*]) for the even order functions.

The group delay responses are similar to the "parent" response of the same order, with a somewhat lower insertion delay at low frequencies and a somewhat higher peak delay at a frequency below the transition f_X, as can be seen in Tables 1, 2 and 3 and Fig 7, before diminishing towards zero at very high frequencies. This will become clearer from examining specific examples.

Even-Order Responses

Even order responses are dealt with first which, like their "parent" Linkwitz-Riley responses, are more forgiving than the odd-order, Butterworth, responses of frequency and phase response errors in the drivers, and have better directional "lobing" properties.

Second Order Response: There are no useful second order functions.

Fourth Order Response: The high-pass and low-pass outputs are combined by addition.

$$F(sT_X)_{\Sigma 4} = \frac{\overbrace{(1 + k^2s^2T_X^2)}^{\text{LOW-PASS}} + \overbrace{s^2T_X^2(k^2 + s^2T_X^2)}^{\text{HIGH-PASS}}}{F(sT_X)_{DEN4}} \qquad - (3)$$

$F(sT_X)_{DEN4}$ is derived by factorising the numerator

$$F(sT_X)_{NUM4} = 1 + 2k^2s^2T_X^2 + s^4T_X^4$$

$$= [\,1 + sT_X\sqrt{\{2(1 - k^2)\}} + s^2T_X^2\,][\,1 - sT_X\sqrt{\{2(1 - k^2)\}} + s^2T_X^2\,] \quad - (4)$$

For the equivalent minimum-phase function of $F(sT)_{DEN4}$ the minus sign of the second term becomes positive, so that

$$F(sT_x)_{DEN4} = [\,1 + x_4sT_X + s^2T_X^2\,]^2 \quad - (5)$$

where $x_4 = \sqrt{[2(1 - k^2)]}$ - (6)

from which the individual low-pass and high-pass functions are

$$F(sT_x)_{LP4} = \frac{1 + k^2s^2T_X^2}{[\,1 + x_4sT_X + s^2T_X^2\,]^2} \quad - (7)$$

and

$$F(sT_x)_{HP4} = \frac{s^2T_X^2\,(k^2 + s^2T_X^2)}{[\,1 + x_4sT_X + s^2T_X^2\,]^2} \quad - (8)$$

and the summed response is the second order all-pass function

$$F(sT_x)_{\Sigma 4} = \frac{1 - x_4sT_X + s^2T_X^2}{1 + x_4sT_X + s^2T_X^2} \quad - (9)$$

When k shrinks to zero, then x_4 becomes $\sqrt{2}$ as in the 2nd order Butterworth function, so that

$F(sT_x)_{LP4}$ and $F(sT_x)_{HP4}$ become 4th order Linkwitz-Riley functions.

The generalised notched responses are plotted in Fig. 1, and the values for the fourth order responses are shown in Table 1 in terms of a crossover frequency f_X of 1000 Hz. The height of the peak amplitude following the notch is dB_{peak}.

In the bottom row of Table 1, figures for group delay response of the Linkwitz-Riley function for $k = 0$ are shown for comparison. Also the frequencies dB_{40}, dB_{35} and dB_{30}, where the Linkwitz-Riley response is down 40dB, 35dB and 30dB respectively, replace f_{peakL}, f_{NL} etc.

It may be seen that steepness of the initial attenuation slope can be traded for magnitude of the following peak.

Table 1. Fourth Order Responses. Peak dB, Out-of-Band Frequencies(Hz) & Group Delays(μs) for various values of k

k^2	dB_{peak}	f_{peakL}	f_{NL}	f_{innerL}	f_X	f_{innerH}	f_{NH}	f_{peakH}	Insertion Delay(μs)	PeakGp Delay	at Hz
1/3	-30.4	414	577	633	1000	1580	1732	2415	368	613	796
1/4	-35.7	355	500	550	1000	1820	2000	2818	390	589	759
1/5	-39.7	316	447	491	1000	2037	2236	3162	403	577	741
0		317	367	425	1000	2352	2726	3154	450	543	644
		dB_{40L}	dB_{35L}	dB_{30L}		dB_{30H}	dB_{35H}	dB_{40H}			

The responses at f_X are -6.02dB for all values of k. The group delay figures for other frequencies of f_X can be scaled inversely with frequency from those quoted above.

Sixth Order Responses: The sixth order functions are derived in a manner similar to the fourth order functions. As in the sixth order Linkwitz-Riley functions, the high-pass and low-pass outputs are combined by subtraction.

$$F(sT_x)_{\Sigma 6} = \frac{\overbrace{(1 + k^2s^2T_X^2)}^{\text{LOW-PASS}} - \overbrace{s^4T_X^4(k^2 + s^2T_X^2)}^{\text{HIGH-PASS}}}{[(1 + sT_X)(1 + x_6 sT_X + s^2T_X^2)]^2} \qquad - (10)$$

where $$x_6 = \sqrt{(1 - k^2)} \qquad - (11)$$

and the summed response is the third order all-pass function

$$F(sT_x)_{\Sigma 6} = \frac{(1 - sT_X)(1 - x_6 sT_X + s^2T_X^2)}{(1 + sT_X)(1 + x_6 sT_X + s^2T_X^2)} \qquad - (12)$$

Table 2. Sixth Order Responses. Peak dB, Out-of-Band Frequencies(Hz) & Group Delays(μs) for various values of k

k^2	dB_{peak}	f_{peakL}	f_{NL}	f_{innerL}	f_X	f_{innerH}	f_{NH}	f_{peakH}	*Insertion Delay*	*Peak Gp Delay(μs)*	*at Hz*
0.5480	-30.0	617	740	779	1000	1283	1351	1622	532	1146	930
0.4653	-35.0	565	682	719	1000	1391	1466	1771	555	1075	915
0.3915	-40.0	515	626	660	1000	1515	1598	1940	567	1025	901
0		465	512	565	1000	1769	1951	2151	637	873	818
		dB_{40L}	dB_{35L}	dB_{30L}		dB_{30H}	dB_{35H}	dB_{40H}			

Eighth Order Responses: Again the eighth order functions are derived in a manner similar to that for the earlier functions. The low-pass and high-pass outputs are combined by addition.

$$F(sT_x)_{\Sigma 8} = \frac{\overbrace{(1 + k^2s^2T_X^2)}^{\text{LOW-PASS}} + \overbrace{s^6T_X^6 (k^2 + s^2T_X^2)}^{\text{HIGH-PASS}}}{[(1 + x_{81}sT_X + s^2T_X^2)(1 + x_{82}sT_X + s^2T_X^2)]^2} \qquad - (13)$$

where $$x_{81} = [\{(4 - k^2) + \sqrt{(8 + k^4)}\} / 2]^{1/2} \qquad - (14)$$

and $$x_{82} = [\{(4 - k^2) - \sqrt{(8 + k^4)}\} / 2]^{1/2} \qquad - (15)$$

and the summed response is the fourth order all-pass function

$$F(sT_x)_{\Sigma 8} = \frac{(1 - x_{81}sT_X + s^2T_X^2)(1 - x_{82}sT_X + s^2T_X^2)}{(1 + x_{81}sT_X + s^2T_X^2)(1 + x_{82}sT_X + s^2T_X^2)} \qquad - (16)$$

Table 3. Eighth Order Responses. Peak dB, Out-of-Band Frequencies(Hz) & Group Delays(µs) for various values of k

k^2	dB_{peak}	f_{peakL}	f_{NL}	f_{innerL}	f_X	f_{innerH}	f_{NH}	f_{peakH}	*Insertion Delay*	*Peak Gp Delay(µs)*	*at Hz*
0.6628	-30.0	719	814	843	1000	1186	1228	1392	710	1761	965
0.5906	-35.0	675	769	797	1000	1255	1301	1483	727	1643	956
0.5224	-40.0	632	723	750	1000	1333	1384	1581	742	1558	949
0		652	606	563	1000	1534	1651	1776	832	1244	888
		dB_{40L}	dB_{35L}	dB_{30L}		dB_{30H}	dB_{35H}	dB_{40H}			

Odd Order Responses

In the same way as the "parent" Butterworth functions, the high-pass and low-pass outputs, which add in quadrature, can be summed either by addition or subtraction for a flat overall response. However, the maximum group delay error, i.e. the difference between the peak and insertion delays, is lower when the 3rd and 7th order outputs are subtracted and when the 5th (and 9th) order outputs are added.

Third Order Response:

$$F(sT_x)_{\Sigma 3} = \frac{\overbrace{(1 + k^2s^2T_X^2)}^{\text{LOW-PASS}} - \overbrace{sT_X(k^2 + s^2T_X^2)}^{\text{HIGH-PASS}}}{[(1 + sT_X)(1 + x_3sT_X + s^2T_X^2)]} \qquad - (17)$$

$F(sT_X)_{DEN3}$ is derived by first factorising the numerator

$$\begin{aligned} F(sT_X)_{NUM3} &= (1 - k^2sT_X + k^2s^2T_X^2 - s^3T_X^3) \\ &= (1 - sT_X)[1 + (1 - k^2)sT_X + s^2T_X^2] \end{aligned}$$

For the equivalent minimum-phase function of the denominator $F(sT_x)_{DEN3}$, the minus sign of the first term becomes positive, so that

$$F(sT_x)_{DEN3} = (1 + sT_X)[(1 + (1 - k^2)sT_X + s^2T_X^2)]$$

Thus

$$F(sT_x)_{\Sigma 3} = \frac{(1 - sT_X)(1 + x_3 sT_X + s^2T_X^2)}{(1 + sT_X)[1 + x_3 sT_X + s^2T_X^2)} = \frac{1 - sT_X}{1 + sT_X} \qquad - (18)$$

where

$$x_3 = 1 - k^2 \qquad - (19)$$

Fifth Order Response:

$$F(sT_x)_{\Sigma 5} = \frac{\overset{\text{LOW-PASS}}{(1 + k^2s^2T_X^2)} + \overset{\text{HIGH-PASS}}{s^3T_X^3(k^2 + s^2T_X^2)}}{(1 + sT_X)(1 + x_{51}sT_X + s^2T_X^2)(1 + x_{52}sT_X + s^2T_X^2)} \qquad - (20)$$

$$= \frac{(1 - x_{52}sT_X + s^2T_X^2)}{(1 + x_{52}sT_X + s^2T_X^2)} \quad \text{(second order all pass)} \qquad - (21)$$

where

$$x_{51} = [-1 + \sqrt{(5 - 4k^2)}] / 2 \qquad - (22)$$

and

$$x_{52} = [+1 + \sqrt{(5 - 4k^2)}] / 2 \qquad - (23)$$

Seventh Order Response:

$$F(sT_x)_{\Sigma 7} = \frac{\overset{\text{LOW-PASS}}{(1 + k^2s^2T_X^2)} - \overset{\text{HIGH-PASS}}{s^5T_X^5(k^2 + s^2T_X^2)}}{(1 + sT_X)(1 + x_{71}sT_X + s^2T_X^2)(1 + x_{72}sT_X + s^2T_X^2)(1 + x_{73}sT_X + s^2T_X^2)} \qquad - (24)$$

$$= \frac{(1 - sT_X)(1 - x_{72}sT_X + s^2T_X^2)}{(1 + sT_X)(1 + x_{72}sT_X + s^2T_X^2)} \quad \text{(third order all pass)} \qquad - (25)$$

The x coefficients of the factors of the seventh order numerator are found from the roots of the equation

$$x_7^3 - x_7^2 - (2 - k^2)x_7 + (1 - k^2) = 0 \qquad - (26)$$

Of the three roots the largest and the smallest magnitudes x_{71} and x_{73} are positive. The middle magnitude root is negative, and its sign is changed to positive to produce x_{72}. Thus for example, when $k^2 = 0.5$, the roots of the

equation are +1.7071, -1.0000 and +0.2929, so the coefficients x_{71} , x_{72} and x_{73} are 1.7071, 1.000 and 0.2929 respectively.

Typical results for the odd order responses are not tabulated because they are believed to be of less interest than the even order responses.

Special Uses of Notched Crossovers

In notched crossovers, the initial slope of attenuation is greatly increased over that of an un-notched filter of the same order, and the minimum out-of-band attenuation can be chosen by the designer, 30dB, 35dB, 40dB or whatever. However the attenuation slope is eventually reduced by 12dB per octave at extreme frequencies. The maximum group delay error is also increased somewhat, though never as much as that for the un-notched filter two orders greater.

These functions should be specially useful when crossovers must be made at frequencies where one or other driver, assumed to be ideal in theory, has an amplitude and phase response that deteriorates rapidly out-of-band, a horn for example near its cut off frequency. Another application is in crossing over to a stereo pair from a single sub-woofer, whose output must be maintained to as high a frequency as possible so as to minimise the size of the higher frequency units, yet not contribute significantly at 250Hz and above where it could muddy localisation.

Realising the Filters

From the designer's point of view, the crossovers are most easily realised as active filters, with each second order factor of the transfer functions realised in the well-known Sallen and Key configuration [*R.P. Sallen & B.L. Key – A practical method of designing RC active filters – Trans. IRE, Vol CT-2, March 1955, pp.74-85*]. An exception is the one factor which provides the notch, with a transfer function of the form, for the low-pass filter,

$$F(sT_X) = \frac{1 + qs(kT_X) + s^2(kT_X)^2}{1 + xsT_X + s^2T_X^2} \qquad - (27)$$

and for the high-pass filter,

$$F(sT_x) = \frac{1 + qs(T_X/k) + s^2(T_X/k)^2}{1 + xsT_X + s^2T_X^2} \qquad - (28)$$

where q is ideally zero and x is the coefficient appropriate to one factor of the desired denominator, e.g. $x_4 = \sqrt{\{2(1 - k^2)\}}$ for the factors of the fourth order crossover.

While q may be made zero in active filters using cancellation techniques, which depend on the balance between component values, quite small values of q can be realised in a Sallen and Key filter that incorporates a bridged T network [*R.P. Sallen & B.L. Key – A practical method of designing RC active filters – Trans. IRE, Vol CT-2, March 1955, pp.74-85, A.N. Thiele – Loudspeakers, enclosures and equalisers – Proc. IREE Aust, Vol. 34, No. 11, November 1973, pp. 425-448*]. Unless a deep notch is really necessary, it will often be sufficient to let the notch "fill up" with a finite value of q. In passive filters, its reciprocal Q (=1/q), the "quality factor" of the reactive elements, has the same effect.

In the sixth order notched crossover, for example, when the height of out-of band peaks are -30dB, -35dB and -40dB, then figures for q of 0.16, 0.14 and 0.10 respectively ensure that the attenuation at the erstwhile notch frequency is no less than at the erstwhile peak and that there is no significant change in response at neighbouring frequencies.

Component values are tabulated in Table 4 for the network of Fig 9 to realise the function

$$F(sT_D) = \frac{1 + x_N sT_N + s^2T_N^2}{1 + x_D sT_D + s^2T_D^2} \qquad - (29)$$

Table 4. Component Values for Sallen & Key Active Filters incorporating a Bridged-T Network, realising Low-Pass and High-Pass Filters for 6th Order Notched Crossovers with f_X = 1 kHz.

$T_X = T_D = 159.2\ \mu s\ :\ (T_N)_{LP} = kT_X\ :\ (T_N)_{HP} = T_X / k$

Both capacitances C1 & C2 are 4.7nF : all resistances in kohms

k	Filter type	x_N	T_N	x_D	T_D	R1a	R1b	R2	R3	R4
0.7403	LP	0.1600	117.8	0.6723	159.2	40.68	2.109	313.4	33.55	∞
(-30dB)	HP	0.1600	215.0	0.6723	159.2	74.23	3.849	571.8	0	693.3
0.6821	LP	0.1400	108.6	0.7313	159.2	29.95	1.709	330.0	34.42	∞
(-35dB)	HP	0.1400	233.3	0.7313	159.2	64.37	3.674	709.2	0	617.0
0.6257	LP	0.1000	99.58	0.7801	159.2	21.37	1.115	423.7	33.22	∞
(-40dB)	HP	0.1000	254.4	0.7801	159.2	54.59	2.847	1082	0	696.3

The second factor of the sixth order transfer function is produced by active high-pass (with numerators of $s^2T_X^2$) or low-pass filters (with numerators of 1) with denominators $1 + x_D sT_D + s^2T_D^2$, where x_D and T_D are as specified, for example, in Table 4.

The low-pass transfer function

$$F(sT_D)_{LP} = \frac{1}{1 + x_D sT_D + s^2T_D^2} \qquad - (30)$$

is realised by the circuit of Fig. 10. First, component values are chosen for C1 and C2. Then the resistances R1 and R2 are defined as the two values of

$$R1, R2 = [T_D/C2][(x_D/2) \pm \sqrt{\{(x_D/2)^2 - (C2/C1)\}}] \qquad - (31)$$

Note that C2/C1 must be less than $(x_D/2)^2$. The nearer the two ratios are to each other, the more nearly equal will be R1 and R2. Preferably R1 is chosen as the larger.

The high-pass transfer function

$$F(sT_D)_{HP} = \frac{s^2T_D^2}{1 + x_D sT_D + s^2T_D^2} \qquad - (32)$$

is realised by the circuit of Fig. 11. C1 and C2 are chosen preferably as equal values C1. Then

$$R1 = (x_D/2)(T_D/C1) \qquad - (33)$$

and

$$R2 = (2/x_D)(T_D/C1) \qquad - (34)$$

There still remain the transfer functions with the denominators

$$F(sT_D) = (1 + sT_D)^2 \qquad - (35)$$

These can be realised simply by cascading two CR sections whose CR products are each T_D. In each filter one CR network could be cascaded with the input, the other with the output. Alternatively the second order functions could be realised in the Sallen and Key filters of Figs 10 & 11 with $x_D = 2$, where for both high-pass and low-pass filters C1 is equal to C2 and R1, equal to R2, is T_D / C1.

In this way, each overall sixth-order transfer function is realised by cascading two or three active stages

$$F(sT_x)_{LP} = \frac{1 + qksT_X + k^2s^2T_X^2}{1 + x_6sT_X + s^2T_X^2} * \frac{1}{1 + x_6sT_X + s^2T_X} * \frac{1}{1 + 2sT_X + s^2T_X^2} \qquad - (36)$$

and

$$F(sT_x)_{HP} = \frac{k^2 + qksT_X + s^2T_X^2}{1 + x_6sT_X + s^2T_X^2} * \frac{s^2T_X^2}{1 + x_6sT_X + s^2T_X^2} * \frac{s^2T_X^2}{1 + 2sT_X + s^2T_X^2} \qquad - (37)$$

and the high and low-frequency drivers are connected in opposite polarities. The coefficient q is of course ideally zero.

The addition of signals to produce a seamless, flat, output assumes of course ideal drivers. If the response errors of the higher frequency, tweeter, driver exceed the propensities for forgiveness of the even order crossover, the middle factor of eqn (37) could be substituted by the equalising transfer function

$$F(sT_x) = \frac{1 + sT_S/Q_T + s^2T_S^2}{1 + x_6 sT_X + s^2T_X^2} \qquad - (38)$$

where $T_S = 1/2\Pi f_S$ and f_S is the resonance frequency of the tweeter and Q_T its total Q. This could be realised in an active filter of the same kind as Fig. 9 [*A.N. Thiele – Loudspeakers, enclosures and equalisers – Proc. IREE Aust, Vol. 34, No. 11, November 1973, pp. 425-448*] When this function is cascaded with the transfer function of the driver

$$F(sT_s) = \frac{s^2T_S^2}{1 + sT_S/Q_T + s^2T_S^2} \qquad - (39)$$

the numerator of eqn (38) cancels with the denominator of eqn (39) to produce the ideal transfer function of the middle factor of eqn. (37).

However, this procedure applies only to crossover functions of sixth or higher order. It must be remembered that the notched crossover, while a sixth order function around the transition frequency, goes to a fourth order slope at extreme frequencies. Thus, because the excursion of a driver rises towards low frequencies at 12dB per octave above its frequency response, its excursion is attenuated only 12dB per octave after such equalisation of a sixth order high-pass notched filter.

If a similar procedure were applied to a tweeter with a 4th order notched crossover function , it would afford incomplete protection against excessive excursion at low frequencies.

Passive Filters

The fourth order passive filters can be realised using the networks of either Fig. 12 or Fig. 13. Either C3L is parallelled across L2L, as in Fig 12(a) - or L3H across C2H as in Fig 12(b) - or L3L is inserted in series with C1L, as in Fig 13(a) - or C3H in series with L1H as in Fig. 13(c). The component values for a low-pass filter of the first kind, in Fig. 12(a), are calculated from the expressions

$$C1L = [\,3(3 - k^2)/4x_4\,][T_X/R_O] \qquad - (40)$$

$$C2L = [\,(1 - 3k^2)/2x_4\,][T_X/R_O] \qquad - (41)$$

$$C3L = [k^2\,(3 - k^2)/\{2x_4(1 - k^2)\}][T_X/R_O] \qquad - (42)$$

$$L1L = [4x_4\,/\,(3 - k^2)\,]T_X R_O \qquad - (43)$$

$$L2L = [2x_4\,(1 - k^2)\,/\,(3 - k^2)\,]T_X R_O \qquad - (44)$$

where $x_4 = \sqrt{[2(1 - k^2)]}$ - (6)

The corresponding high-pass components are calculated from the low-pass components, in all cases, using the generalised expressions

$$CnH = T_X^2\,/\,LnL \qquad - (45) \qquad \text{and} \qquad LnH = T_X^2\,/CnL \qquad - (46)$$

The resulting high-pass filter, Fig 12(b), can additionally be adapted to sensitivity control using an auto-transformer [*D.E.L. Shorter – A survey of performance criteria and design considerations for high quality monitoring loudspeakers – Proc. IEE 105 Part B, 24 November 1958, pp. 607-622 also reprinted and in Loudspeakers, An Anthology, Vol 1 – Vol 25 (1953-1977), ed. R.E. Cooke – Audio Engineering Society, inc, New York, October 1978, pp. 56-71, A.N. Thiele – An air cored auto-transformer (to be published)*]. However that network requires high values in the Π network of inductances transformed from the Π network of capacitances C1L, C2L and C3L, especially L2H, transformed from the small values of C2L. In fact, when k^2 is 1/3, then C2 is

zero and L2H goes to infinity. They are more easily realised from a Δ-Y transformation into the network of Fig. 12(c), where

$$C1H = [(3 - k^2)/4x_4][T_X / R_O] \quad - (47)$$

$$C2H = [(3 - k^2)/2x_4 (1 - k^2)][T_X / R_O] \quad - (48)$$

$$L1H' = [4x_4 (1 - k^2)(1 - 3k^2)/(3 - k^2)^2]T_X R_O \quad - (49)$$

$$L2H' = [6x_4 (1 - k^2)/(3 - k^2)]T_X R_O \quad - (50)$$

$$L3H' = [4x_4 k^2/(3 - k^2)]T_X R_O \quad - (51)$$

The set of three inductances can be realised either individually or, more conveniently, from two inductors

$$L1H' + L2H' = [2x_4 (1 - k^2)(11 - 9k^2)/(3 - k^2)^2]T_X R_O \quad - (52)$$

$$L1H' + L3H' = [4x_4 (1 - k^2 + 2k^4)/(3 - k^2)^2]T_X R_O \quad - (53)$$

which are wound separately and then coupled together *in series opposition* so that their mutual inductance is L1H', i.e. the coupling coefficient between them is

$$| k_{COUPLING} | = [2 (1 - k^2)(1 - 3k^2)^2 / (1 - k^2 + 2k^4)(11 - 9k^2)]^{1/2} \quad - (54)$$

The resulting filter, Fig. 12(d), may look rather strange but is eminently practical. The mutual inductance is realised in L1H' rather than L3H' because that procedure leads to smaller sum inductances L1H' + L2H' and L1H' + L3H' over the range of k^2 between 0.333 and 0.157 that is of most practical use. The coupling coefficients $k_{COUPLING}$ are small enough to be easily achieved. To produce the required coupling, the spacing between the two coils is adjusted until their inductance, measured end to end, is L2H' + L3H'. The procedure realises all the inductances in the one unit, which can include an air-cored auto-transformer [*A.N. Thiele - An air cored auto-transformer (to be published)*] and

is easily mounted without any worry about stray couplings between individual inductors

In the alternative realisations of the second kind, in Fig 13(a), the low-pass components are

$$C1L = [9(1 - k^2)/4x_4][T_X / R_O] \quad - (55)$$

$$C2L = T_X / 2x_4 R_O \quad - (56)$$

$$L1L = 4x_4 T_X R_O / 3 \quad - (57)$$

$$L2L = 2x_4 T_X R_O / 3 \quad - (58)$$

$$L3L = [4x_4 k^2 / 9 (1 - k^2)] T_X R_O \quad - (59)$$

This second version of the low-pass filter, Fig. 13(a) again needs three inductances, and can again be produced by winding one coil to a value of L1L + L3L another with a value of L2L + L3L and coupling them together *in series opposition* to produce L3L as the mutual inductance between them, as in Fig. 13(b). This is again produced by varying their coupling until

$$|k_{COUPLING}| = [2k^4/(3 - 2k^2)(3 - k^2)]^{1/2} \quad - (60)$$

and the inductance end-to-end reads L1L + L2L. Again there is only the one component to mount and no further need to position the inductors to avoid stray coupling. Also in this case, because the mutual inductance L3L is free of a resistive component, the filter is capable of a better null.

The high-pass component values for Fig 13(c) are again derived from the low-pass values via eqns (45) and (46).

Each version has its uses. In the first kind, Fig. 12(a), C2L goes to zero when $k^2 = 1/3$, i.e. when the following peak height is -30.4dB. Larger values of k

require a negative mutual inductance, but are unlikely to be needed in practice, with following peak heights higher than -30dB. The high pass filter of the second kind, Fig 13(c) is less desirable than the first kind. It requires three capacitors, one of which C3 is comparatively large.

Component values for a crossover frequency of 1000 Hz and a terminating resistance of 10 ohms are presented in Table 5 for all four realisations of each of the three fourth order versions, with following peaks of approximately -30dB, -35dB and -40dB.

Table 5. Fourth Order Passive Notched Crossovers. Component Values for f_X = 1000 Hz and R_0 = 10 ohms

Low-Pass Filter (with C3 in parallel with L2)

k	*L1(μH)*	*C1(μF*	*L2(μF)*	*C3(μF)*	*C2(μF)*
0.5774	2757	27.57	919	9.189	0
0.5000	2835	26.80	1063	5.956	1.624
0.4472	2876	26.42	1150	4.404	2.516
0	3001	25.32	1501	0	5.627

High-Pass Filter (with L1 L2 & L3 in network around C2)

k	*C1(μF)*	*L1(μH)*	*C2(μF)*	*L3(μH)*	*L2(μH)*	$k_{COUPLING}$
0.577	9.189	0	27.57	918.9	2757	0
0.5000	8.934	193.3	23.82	708.8	3190	0.1107
0.4472	8.808	328.7	22.02	575.2	3451	0.1778
0	8.440	1000.3	16.88	0	4502	0.4264

Low-Pass Filter (with L3 in series with C1)

k	*L1(μH)*	*C1(μF)*	*L3(μH)*	*L2(μH)*	*C2(μF)*	$k_{COUPLING}$
0.5774	2450	20.68	408.4	1225	6.892	0.1890
0.5000	2599	21.93	288.8	1299	6.497	0.1348
0.4472	2684	22.65	223.7	1342	6.291	0.1048
0	3001	25.32	0	1501	5.627	0

High-Pass Filter (with C3 in series with L1)

k	*C1(μF)*	*L1(μH)*	*C3(μF)*	*C2(μF)*	*L2(μF)*
0.5774	10.34	1225	62.02	20.68	3676
0.5000	9.746	1155	87.72	19.49	3898

0.4472	9.437	1118	113.2	18.87	4026
0	8.440	1000	∞	16.88	4502

Input Impedance

The input impedances of the passive filters are identical for the two kinds of realisations in Figs 12 and 13.

The input impedances of passive crossover filters are best assessed by splitting them into parallel components of resistance R and reactance X, that of the low-pass filter into R_{LP} and X_{LP} and that of the high-pass filter into R_{HP} and X_{HP}. The resistances R_{LP} and R_{HP} vary in inverse proportion to their responses or, more precisely, to the powers that reach their outputs.

When the inputs of the two filters are connected in parallel, the resulting joint input resistance is

$$R_{IN} = R_{LP}R_{HP} / (R_{LP} + R_{HP}) \qquad - (61)$$

while the joint input reactance

$$X_{IN} = X_{LP}X_{HP} / (X_{LP} + X_{HP}) \qquad - (62)$$

Then $$Z_{IN} = 1 / \surd[(1/R_{IN}^2) + (1/X_{IN}^2)] \qquad - (63)$$

Values of these quantities, for a notched crossover with $k^2 = 1/3$, i.e. $k = 0.5774$, derived as in Appendix A, are shown in Table 6.

Table 6. Input Impedance Z_{IN} and Parallel Components of Resistance R and Reactance X (ohms) of Fourth Order Notched Low Pass and High Pass Filters

Crossover frequency f_X = 1000 Hz, Notch ratio k = 0.5774, Terminating Resistance = 10 ohms

f(Hz)	316	398	501	631	794	1000	1259	1585	1995	2512	3162

$R_{LP}(\Omega)$	9.5	9.4	9.6	10.6	15.3	40.0	270.9	12.0K	18.8K	11.0K	16.3K
$X_{LP}(\Omega)$	42.1	29.6	20.2	14.0	10.9	11.6	16.2	23.6	31.9	41.5	53.1
$R_{HP}(\Omega)$	16.3K	11.0K	18.8K	12.0K	270.9	40.0	15.3	10.6	9.6	9.4	9.5
$X_{HP}(\Omega)$	-53.1	-41.5	-31.9	-23.6	-16.2	-11.6	-10.9	-14.0	-20.2	-29.6	-42.1
$R_{LP//HP}(\Omega)$	9.5	9.4	9.6	10.6	14.5	20.0	14.5	10.6	9.6	9.4	9.5
$X_{LP//HP}(\Omega)$	203.1	103.4	55.3	34.3	33.6	∞	-33.6	-34.3	-55.3	-103.4	203.1
$Z_{IN}(\Omega)$	9.5	9.4	9.4	10.1	13.3	20.0	13.3	10.1	9.4	9.4	9.5

They are also plotted in Fig 14, where they can be compared with similar plots in Fig 15, for a Butterworth crossover, and Fig 16, for a Linkwitz-Riley crossover which, as we have seen already, may be considered as a notched crossover with k = 0.

In Fig. 14 solid curves show R_{HP} (top left), R_{LP} (top right) and R_{IN} (lowest middle), and dashed curves show X_{LP} (lowest on left), X_{HP} (middle) and X_{IN} (upper on left). X_{LP} is +ve at all frequencies and X_{HP} is -ve at all frequencies, so $-X_{HP}$ is plotted at all frequencies. X_{IN} is +ve at low frequencies and -ve at high frequencies, so $-X_{IN}$ is plotted at high frequencies.

In Fig. 15 solid curves show R_{HP} (top left) and R_{LP} (top right) and dashed curves show X_{LP} for low-pass filter. X_{HP} has identical magnitude but -ve sign. R_{IN} = 1 at all frequencies and X_{IN} is infinite at all frequencies. Therefore neither is plotted.

In Fig. 16 solid curves show R_{HP} (top left), R_{LP} (top right) and R_{IN} (lowest middle), and dashed curves show X_{LP} (lowest on left), X_{HP} (middle) and X_{IN} (upper on left). X_{LP} is +ve at all frequencies and X_{HP} is -ve at all frequencies, so $-X_{HP}$ is plotted at all frequencies. X_{IN} is +ve at low frequencies and -ve at high frequencies, so $-X_{IN}$ is plotted at high frequencies.

In Fig 15, the normalised input resistance R_{IN} for the Butterworth crossover is 1 at all frequencies, so there is no point in plotting it. Since $X_{LP} = -X_{HP}$, their sum $X_{LP} + X_{HP}$ is zero and therefore X_{IN} is infinite at all frequencies. This applies only to Butterworth crossovers, and then only when both filters are terminated in the same resistance R_0. However if, for example, $X_{LP} = -1.5X_{HP}$, their combined X_{IN} would be $3X_{HP}$, i.e. $-2X_{LP}$, and if $R_{LP} = 1.5R_{LP}$ then $R_{IN} = 0.6R_{HP}$. In both cases R_{IN} and X_{IN} would vary with frequency.

The input impedance of the notched and Linkwitz-Riley crossovers varies in a rather more complicated manner. The resistive and reactive components for the high-pass and low-pass filters are symmetrical in frequency in that their magnitudes for the high-pass filter at any frequency nf_X are the same as those for the low-pass filter at the frequency f_X/n. The sign of the reactive components is always negative for the high-pass filter and always positive for the low-pass filter but their magnitudes are equal, and cancel in parallel, only at the transition frequency. At other frequencies, the magnitude of their combined reactance is never less than 3 times the nominal, terminating, impedance R_0. The resistive component of each filter is $4R_0$ at the transition frequency, (the two in parallel present $2R_0$), rising rapidly at frequencies outside the pass-band.

In the notched crossover filters, the resistive component diminishes within the pass-band through R_0 at the notch frequency of the other filter to a minimum, never lower than $0.94R_0$, before returning to R_0 at extreme frequencies. The reason is that, as explained earlier, each filter must, at frequencies in its pass-band beyond the notch of the other filter, deliver a power a little greater (0.27dB maximum) than its input so as to maintain a flat combined output. To produce more power from a low (virtually zero) impedance source, the filter must present a lower resistance component of input impedance.

Table 6 and Figs 14, 15 & 16 show that, in all types, the resistance component tends to dominate the input impedance. For example, if R_{IN} is 10 Ω and X_{IN} is 30 Ω, then Z_{IN} is 9.49 Ω. Nevertheless the presence of shunt reactance and its possible effect on the driving amplifier should always be kept in mind.

Like most passive crossovers, these networks require ideally an accurate and purely resistive termination. Unless the driver presents a good approximation to such a resistance, its input terminals will need to be shunted by an appropriate impedance correcting network[*A.N. Thiele – Optimum passive loudspeaker dividing networks – Proc. IREE Aust, Vol 36, No 7, July 1975, pp. 220-224*].

The notched crossover systems, especially those using even order functions, offer improvements in performance, particularly when rapid attenuation is needed close to the transition frequency. Although their performance in lobing with non-coincident drivers has not been examined specifically, it is expected to be similar to that of the Linkwitz-Riley crossovers, because their two outputs maintain a constant zero phase difference across the transition.

The passive filters that utilise coupling between inductors also offer convenience in realisation and in mounting in the cabinet as a single unit.

The odd-order functions, whose high-pass and low-pass outputs add in quadrature, have been included for completeness, though they would seem to be of less general interest than those of even order.

NON ELECTRICAL DOMAINS

The present invention is readily applied to domains other than electrical domains because there exists a well understood correspondence between quantities such as current, voltage, capacitance, inductance and resistance in the electrical domain and counterparts thereof in the other domains. Table 7 shows the correspondence between analogous quantities in the electrical, mechanical and acoustical domains. The quantities are analogous because their differential equations of motion are mathematically the same.

Table 7

Electrical		Mechanical		Acoustical	
Current	Amps	Velocity	m/sec	Volume velocity	m^3/sec
Voltage	Volts	Force	N	Pressure	N/ m^2 or Pa
Capacitance	Farads	Mass	kg	Acoustical compliance	m^5/N
Inductance	Henrys	Mechanical compliance	m/N	Acoustical mass	kg/m^4
Resistance	Ohms	Mechanical responsiveness	m/Nsec	Acoustical resistance	Nsec/m^5

Figure 17 shows an example of a filter realized in an acoustical domain which is a direct analog of the low pass and high pass filters shown in Figs 13a and 13c. In Fig. 17 C1, C2 and C4 are vented chambers, C3 and C5 are flexible membranes, D1 to D5 are ducts which may be of any cross-sectional shape but in this example will be assumed to be circular, and R1 to R2 are sieves which dissipate energy from fluids passing through them.

The input is pressure generator P1. The low frequency output is pressure at sensor V1 and the high frequency output is pressure at sensor V2.

Assume that the crossover frequency f_x is 10 Hz. Then $T_x = 1/(2\pi f_x) = 15.9$mS.

Assume that dB_{peak} in Fig 1 is set at –40dB, then according to Table 1, $k^2 = 0.2$, therefore $k = 0.447$.

Assume that the sieves R1 and R2 each have acoustic resistance of 2000 NS/m^5.

According to Equation 6, $x_4 = \sqrt{[2(1-k^2)]} = 1.265$

Using Equations 55 to 59 the following values are obtained.
C1L = 11uF, C2L = 3.1uF, L1L = 53H, L2L = 26H, L3L = 4.4H

Duct D1 corresponds to L1L and has a corresponding acoustic mass of 53kg/m^4.

Duct D2 corresponds to L3L and has a corresponding acoustic mass of 4.4kg/m^4.

Duct D3 corresponds to L2L and has a corresponding acoustic mass of 26kg/m^4.

Chamber C1 corresponds to C1L and has an acoustic compliance of 11×10^{-6} m^5/N.

Chamber C3 corresponds to C2L and has an acoustic compliance of 3.1×10^{-6} m^5/N.

Using Equations 45 and 46 the remaining values can be defined as follows:

Duct D4 corresponds to L1H and has an acoustic mass of 22kg/m^4.

Duct D5 corresponds [illegible] acoustic mass of 81kg/m^4.

Chamber C4 corresponds to C3H and has an acoustic compliance of 57×10^{-6}m^5/N.

Membrane C3 corresponds to C1H and has an acoustic compliance of 4.7×10^{-6}m/N.

Membrane C5 corresponds to C2H and has an acoustic compliance of 9.4×10^{-6}m/N.

These values can be converted to physical dimensions using the conversions familiar to artisans in the acoustic domain. For example, assuming an air density (ρ_0) of 1.18kg/m^3 and speed of sound in air (c) of 345 m/S, the length to cross sectional area ratios of the ducts in SI units will be acoustic mass divided by 1.18. Assuming a duct diameter of 200mm the length of ducts will be as follows: Duct D1 1.4m, duct D2 120mm, duct D3 710mm, duct D4 600mm, duct D5 2.1m. The chamber volumes will be the acoustic compliance multiplied by $\rho_0 c^2$, which works out to 1.6m^3 for chamber C1, 0.44m^3 for chamber C2, 1.3m^3 for chamber C4. The membrane characteristics of C3 and C5 are such that the volume displaced divided by the pressure exerted on the membrane provides the values previously indicated.

Finally, it is to be understood that various alterations, modifications and/or additions may be introduced into the constructions and arrangements of parts previously described without departing from the spirit or ambit of the invention.

Appendix

Parameters for Input Impedance of Passive Fourth Order Notched Crossover Filters

The input impedances Z_{LP} and Z_{HP} of the passive low-pass and high-pass filters and their parallel combination Z_{IN} are best considered by partitioning them into parallel components of resistance R_{LP}, R_{HP}, R_{IN} and reactance X_{LP}, X_{HP}, X_{IN}, whose values are derived below

$$R_{LP} = R_O \left[\frac{1 - 2k^2a^2 + a^4}{1 - k^2a^2} \right]^2 \qquad \text{- (A1)}$$

$$R_{LP} = R_O \left[\frac{1 - 2k^2a^2 + a^4}{k^2a^2 - a^4} \right]^2 \qquad \text{- (A2)}$$

where the normalised frequency variable $a = \omega T_X = f/f_X$. The expressions for the resistive components are, not surprisingly, inversely proportional to the squared magnitudes of the frequency responses of the filters, i.e. to the power that they absorb from the input. The resistive component of their parallel combination is

$$R_{LP} = R_O \frac{(1 - 2k^2a^2 + a^4)^2}{1 - 2k^2a^2 + 2k^4a^4 - 2k^2a^6 + a^8} \qquad \text{- (A3)}$$

These are shown in the solid curves of Fig 14. The reactive components, shown in the dashed curves of Fig 14, are

$$X_{LP} = \frac{4\sqrt{(2 - 2k^2)}\, R_O\ (1 - 2k^2a^2 + a^4)^2}{(5 - 7k^2)a + (7 - 11k^2 + 10k^4)a^3 + (1 - 13k^2 + 6k^4)a^5 + (3 - k^2)a^7} \qquad \text{- (A4)}$$

$$X_{HP} = \frac{4\sqrt{(2 - 2k^2)}\, R_O\ (1 - 2k^2a^2 + a^4)^2}{(3 - k^2)a + (1 - 13k^2 + 6k^4)a^3 + (7 - 11k^2 + 10k^4)a^5 + (5 - 7k^2)a^7} \quad \text{- (A5)}$$

While X_{LP} is positive at all frequencies, X_{HP} is negative at all frequencies. Thus, because the y axis of Fig 14 must be plotted on a logarithmic scale to accommodate the great variations in magnitude , X_{HP} is plotted there as - X_{HP} .

$$X_{IN} = \frac{2\sqrt{(2 - 2k^2)}\, R_O\ (1 - 2k^2a^2 + a^4)^2}{(1 - 3k^2)(a - a^7) + (3 + k^2 + 2k^4)(a^3 - a^5)} \quad \text{- (A6)}$$

Because X_{IN} is positive at all frequencies below f_X, and negative at all frequencies above f_X, it is plotted in Fig 14 as its magnitude $|X_{IN}|$. The combined input impedance Z_{IN} is less than R_{IN} by so small a margin that its plot would have needlessly cluttered Fig 14. It is therefore omitted.